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Modeling Terminal Velocity

Purpose / Rationale

The main goal for this lesson is to reinforce the connection between position, velocity, and acceleration by investigating a natural phenomenon, air resistance. Specifically we want to describe mathematically how air resistance forces are related to velocity. Students will be required to interpret graphs and use quantities derived from them to measure the air resistance forces. This can be a confusing addition to forces and motion, but important because of its real world applications. This lesson is intended for students who have a comfortable grasp on forces and motions already.

Safety Issues

Limiting mass of the coffee filters will help lower the height necessary to find terminal velocity.

SOLS

A.8 The student, given a situation in a real-world context, will analyze a relation to determine whether a direct or inverse variation exists, and represent a direct variation algebraically and graphically and an inverse variation algebraically.

A.6 The student will graph linear equations... in two variables, including
a) Determining the slope of a line when given... two points on the line. Slope will be described as rate of change

PH.1 The student will plan and conduct investigations in which a) the components of a system are designed b) instruments are selected are used to extend observations and measurement of mass, motion c) information is recorded and presented in an organized format d) metric units are used in measurements and calculations e) the limitations of the experimental apparatus and design are recognized h) appropriate technology including computers, graphing calculators, and probeware is used gathering and analyzing data and communicating results.

PH.2 The student will investigate and understand how to analyze and interpret data. Key concepts include a) a description of a physical problem in translated into a mathematical statement in order to find a solution b) relationships between physical quantities are determined using the shape of a curve passing through experimentally obtained data c) the slope of a linear relationship is calculated and includes appropriate units d) interpolated, extrapolated and analyzed trends are used to make predictions.

PH.4 The students will investigate and understand how application of physics affect the world. Key concepts include a) examples for the real world.

PH.5 The students will investigate and understand the interrelationships among mass, distance, force and time through mathematical and experiment process. Key concepts include a) linear motion d) Newton's laws of motion

Materials and Resources

- Coffee Filters
- Motion Detector
- Data Acquisition from Vernier Software
- Data collection sheets

Engage:

Show them a video of a sky diver. Ask students why two skydivers of the same mass, one who jumps from 1 km up, and another that jumps from 10km up, both reach the a speed of around 50 m/s but no faster. Review free fall briefly. Students should recall that in the absence of air resistance, all objects continue to accelerate at the same rate, independent of mass. Explain that both sky divers reached what is called the terminal velocity. Falling objects approach a maximum constant speed because an upward force of air resistance that acts to balance the opposite gravitational force. Ask students about their ideas of what might affect this force of air resistance.

Explore:

Ask students to construct a free body diagram of a falling coffee filter. Propose a model for the force of air resistance, $F_r = cv$. Guide the students to realize that at the terminal velocity (i.e. constant velocity) there is not acceleration, therefore, there are no net force, meaning the force of air resistance is equal and opposite to the force of gravity, $mg = cv_{terminal}$, or $m = (c/g)\ v_{terminal}$. In this model the terminal velocity is *linearly proportional* to the mass of the falling object. Explain that the goal of the lab is to determine the constant 'c' that connects mass and terminal velocity. Before beginning the lab, show students examples of 'good' and 'bad' trials. Ask students to think of what are plausible and physically impossible measurements in the experiment.

After determining the definition for terminal velocity, ask how terminal velocity affects the relationship between position and time. How would this relationship look on the graph of position over time? Ask students to identify where they think that the terminal velocity occurs on the given example graph.

Introduce the "Air Resistance Lab" and hand out activity sheets. Tell students that they will be using the Vernier Motion Detector 2 to gather data points describing the position of a coffee filter versus time. Since we are concerned about the relationship between terminal velocity and mass, and we are asking if there is a relationship between the two, the students will need to determine the terminal velocity from their position versus time data. Introduce idea that you will only be able to gather data points of position.

Students record the mass of the coffee filter(s) on each data run. (e.g. One coffee filter has a mass of 5 g). Student will test up to five coffee filters. You will use the Vernier Motion Detector 2 to measure the distance from the probe that your coffee filter is at a given time as it falls to the ground. They will record the data from the chart on their Vernier LabQuest device onto the graph paper given on their Data Collection Sheet. Students will need to figure out a way to calculate the terminal velocity. (By calculating the slope of the linear region of the position graph)

Explain:

Reinforce the fact that the velocity is the rate of change of position, and what the interpretations of the graph really mean. Ask students if they observe any patterns in the relationship between velocity and mass after they have graphed the data. After class discussion about their observations, solidify that velocity and mass are proportional.

The relationship discovered is suitable for slowing moving falling objects in a fluid (coffee filters in air, pebbles in water), but a fast moving object moving through a fluid (a skydiver in air, bullet in water) follows a different model of air resistance. Show a quick video of a bullet rapidly decelerating in water (http://www.youtube.com/watch?v=TcyEWT2O550) Explain how fast objects create low pressure pockets in the fluid in their wake. These pockets invite more air to move around and in front of the falling object. A ppt could assist in the explanation.

Elaborate:

Students will be given sample data from a skydiver to attempt to discover the new model, knowing that it still depends on velocity and mass. Students are shown that plotting the data in the same m vs. v manner does not produce a straight line. Students are then asked to plot m vs. v^2 and to explain why what this shows, and why this model of air resistance is more suitable for the data. It is also possible to further expand the discussion by using air resistance as a brief introduction to the concept of a differential equation in the sense that a falling object's acceleration (second derivative) is related to its velocity (first derivative).

Evaluation:

For homework student will be given an activity sheet that asks content questions about position versus time and velocity versus time graphs. Also included is an activity that requires them to take given sample data and choose which model is best suited for the data.

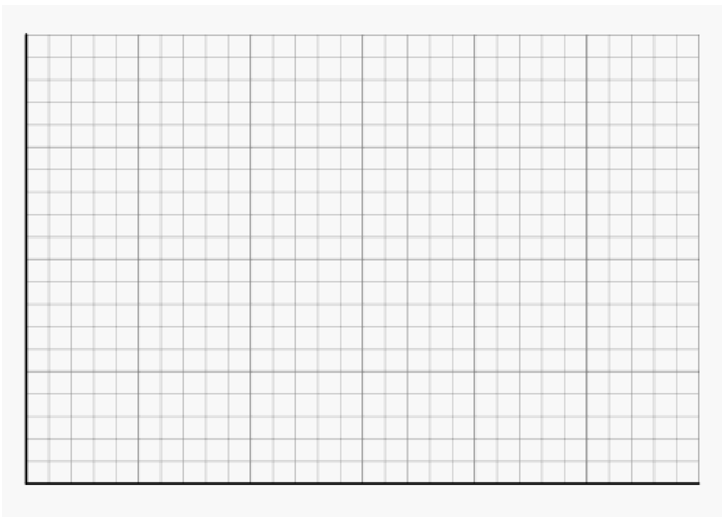
Names:

Data Collection Sheet

First Trial:

One coffee filter weighs _____________ (1a).

Graph the position vs. time data, and find the terminal velocity:



Terminal velocity (1b): ___________________

Second Trial:

Two coffee filters weigh _____________ (2a).

Graph the position vs. time data, and find the terminal velocity:

Terminal velocity (2b):_____________________

Third Trial:

Three coffee filters weigh ____________ (3a).

Graph the position vs. time data, and find the terminal velocity:

Terminal velocity (3b):___________________

Fourth Trial:

Four coffee filters weigh ____________ (4a).

Graph the position vs. time data, and find the terminal velocity:

Terminal velocity (4b):___________________

Fifth Trial:

Five coffee filters weigh ____________ (5a).

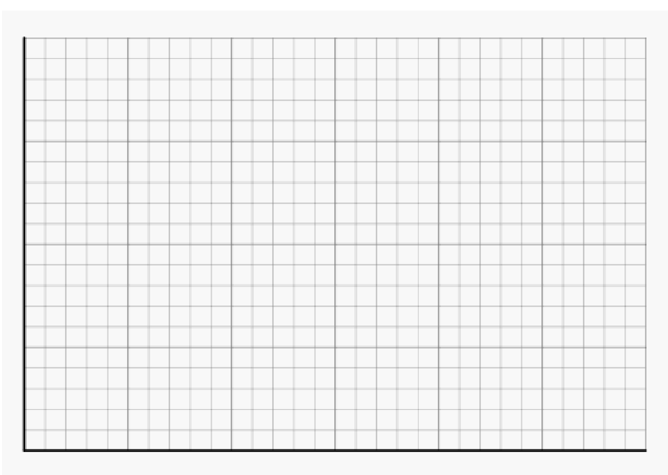
Graph the position vs. time data, and find the terminal velocity:

Terminal velocity (5b):___________________

Now rerecord your data in the following table.

Mass	Velocity
(1a)	(1b)
(2a)	(2b)
(3a)	(3b)
(4a)	(4b)
(5a)	(5b)

Graph velocity vs. mass data:



Measured Proportionality constant : ____________ (Don't forget its unit)

Write a conclusion about what you observe in the graph of the velocity versus mass.

Name:

A Different Model of Air Resistance.

Crash test dummies of different masses were tossed out of a high flying plane for the purpose of estimating a model of a falling human's air resistance. You are given the masses and terminal velocities for each of the dummies. First plot velocity vs. mass. Include a titles and correct scaling.

Mass (kg)	10	50	75	100	125	250	500	1000
Terminal Velocity (m/s)	20.4	45.6	55.9	64.5	72.1	102.0	144.3	204.1
$V_{terminal}^2$ (m^2/s^2)								

You should notice that it is more difficult to connect the data points in a straight line. This is because in this model of air resistance, one for faster moving forces, mass is proportional to the square of the velocity. Fill in the data table column. Plot mass vs. v^2. Calculate the slope of the line to discover the constant relating mass and the square of the terminal velocity

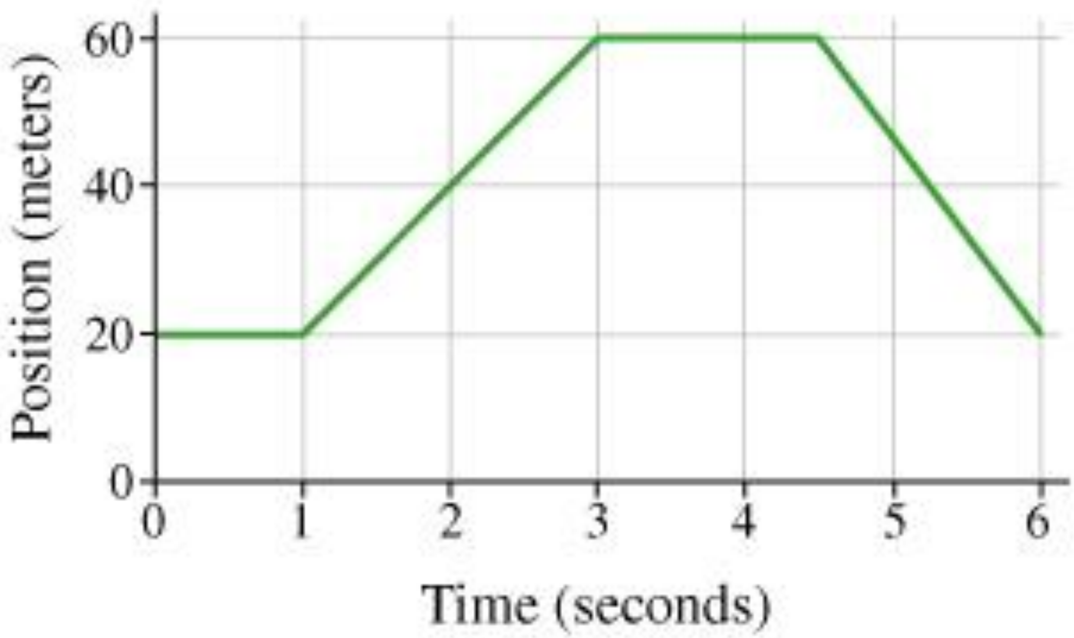
Proportionality constant : ____________ (Don't forget its unit)

According to this new model of air resistance. What would be your terminal velocity?

In your own words, why do you think faster moving objects follow a different model of air resistance?

Name:

Homework Part 1: x vs. t Plot Review



1. Using the following graph, find:

   a) the average velocity from t=0-4 s.

   b) the instantaneous velocity at t=2 s

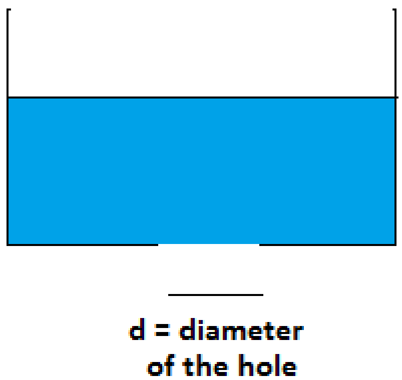
   c) the instantaneous acceleration at t=5 s

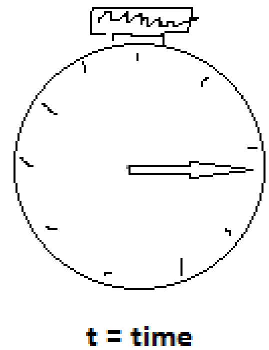
2. How is this plot unrealistic for the actual motion of an object? Think about the instantaneous velocity at t =3s and t =4.5s.

Name:

Homework Part 2: Discovering a Relationship

Let's say you wanted to know how much time it takes for as certain amount of water to empty out of a container through a hole in its bottom. If the amount of water is fixed, the time will only depend on the size of the hole. What we need is a relationship that relates time to the size of the hole. This kind of knowledge would be important to a sink manufacturer for instance. Provided is sample data of different diameter wholes and the corresponding times.





Diameter (cm)	1.5	2	3	5
Time (s)	73.0	41.2	18.4	6.8

It is your job to decide what model best fits this set of data. Consider the physical situation. How is the area of the whole related to the diameter? As the diameter approaches infinity what does time approach? As the diameter approaches zero what does time approach?

This will likely take several attempts and some guessing and checking but the correct model should allow for the data to be graphed linearly and a proportionality constant to be calculated.

What model relating time and diameter did you settle on?

What proportionality constant relates time and diameter in this model?

Assessment Rubric:

1 - Needs Improvement	2 - Good	3 - Excellent
Students measure mass and velocity but does not graph relationship or seem to understand connection between two quantities.	Students measure and identify relationship between mass and velocity but exhibits confusion in interpretation.	Students measure and identify relationship between mass and velocity and exhibits understanding in the relationship between mass and velocity
Student graphs show little relationship to reality and large amounts of extraneous data are left in the graphs	Student graphs show resemblance to real physical relationship but some extraneous data remains.	Student graph shows great relationship to real physical relationship and little to no extraneous data remains
Student graphs are sketched, not legible, and difficult to read or are very poorly or not labelled.	Student graphs are legible and readable, but appear rushed and have errors (lack of labelling, etc.)	Student graphs are clean, readable, and have few to no errors evident.
Conclusions do not reflect understanding about linear relationships	Conclusions do reflect understanding about linear relationships but with small errors	Conclusions do reflect understanding about linear relationships with practically no errors.

	Latest		
	Time (s)	Position (m)	Velocity (m/s)
1	0.05	0.184	-0.027
2	0.10	0.181	-0.010
3	0.15	0.182	0.007
4	0.20	0.183	0.011
5	0.25	0.184	0.004
6	0.30	0.183	0.015
7	0.35	0.182	0.115
8	0.40	0.185	0.407
9	0.45	0.219	0.762
10	0.50	0.269	0.976
11	0.55	0.323	1.046
12	0.60	0.373	1.093
13	0.65	0.437	1.060
14	0.70	0.475	1.095
15	0.75	0.546	1.144
16	0.80	0.594	1.133
17	0.85	0.653	1.227
18	0.90	0.717	1.305
19	0.95	0.787	1.313
20	1.00	0.852	1.253
21	1.05	0.911	1.252
22	1.10	0.967	1.411
23	1.15	1.048	1.586
24	1.20	1.156	1.200
25	1.25	1.184	0.511
26	1.30	1.187	0.164
27	1.35	1.187	0.033
28	1.40	1.187	-0.003
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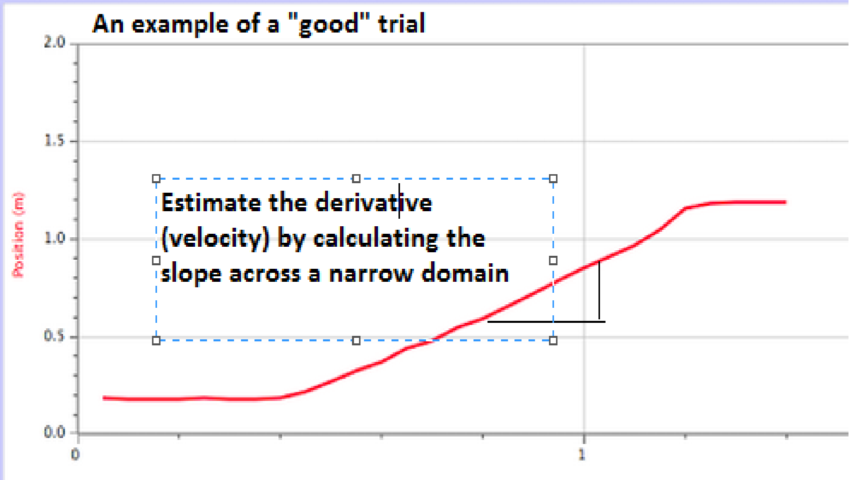
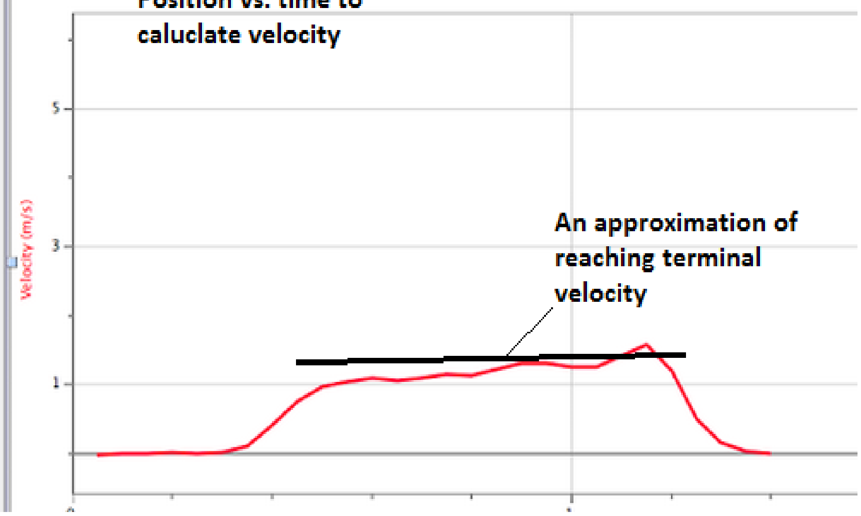


Position vs. time to caluclate velocity



Latest		
Time (s)	Position (m)	Velocity (m/s)
0.05	0.183	-0.002
0.10	0.184	-0.018
0.15	0.180	-0.012
0.20	0.182	0.008
0.25	0.182	0.008
0.30	0.183	0.002
0.35	0.182	-0.011
0.40	0.181	-0.005
0.45	0.182	-0.004
0.50	0.180	0.005
0.55	0.183	0.018
0.60	0.184	0.004
0.65	0.182	0.002
0.70	0.184	-0.002
0.75	0.182	-0.014
0.80	0.181	-0.002
0.85	0.182	0.006
0.90	0.182	0.018
0.95	0.186	0.006
1.00	0.183	-0.018
1.05	0.182	-0.007
1.10	0.182	0.025
1.15	0.182	0.136
1.20	0.185	0.459
1.25	0.224	0.867
1.30	0.278	1.142
1.35	0.345	1.215
1.40	0.413	0.998
1.45	0.459	0.520
1.50	0.459	0.140
1.55	0.459	0.028
1.60	0.459	0.003
1.65	0.459	0.000
1.70	0.459	0.000
1.75	0.459	-0.001
1.80	0.459	-0.001
1.85	0.459	0.001
1.90	0.459	0.002
1.95	0.459	-0.001
2.00	0.459	-0.002
2.05	0.459	0.000
2.10	0.459	0.003
2.15	0.459	0.002
2.20	0.459	0.000
2.25	0.459	-0.001